

# Chapter 12

## Logarithmic Links: Logarithmic, Exponential and Power Functions

### 12.1 The Definition of e

#### Example 1

Consider the exponential function

$$f(x) = \left(1 + \frac{1}{x}\right)^x$$

What happens to  $f(x)$  as  $x$  gets very large?

Type the function into **Y=** (see Fig. 12.1).

#### 12.1.1 Use TABLE to Find values

Press **2nd** **TblSet**. Put the independent variable in ASK mode (see Fig. 12.2).

Press **2nd** **TABLE**.

Type 100 **ENTER** (see Fig. 12.3). The calculator will only find the values of Y1 as you enter  $x$  values.

Continue to enter powers of 10 values as in Figure 12.4.

#### Interpretation:

As  $x \rightarrow +\infty$ ,  $f(x) \rightarrow 2.718\ 281\ 8271\dots$ . We say that the function approaches a *limit*, or gets close to a number value.

#### Example 2

Compare the limit value of  $f(x)$  to the calculator value of  $e$ .

Press **2nd** **QUIT** **CLEAR**. Store  $10^9$  to  $x$ .

Type Y1 and  $e^1$  (see Fig. 12.5).

**Note:**  $e$  is located above **LN** and you must supply a power.

Notice that are nearly the same. The value of  $e$  is an irrational number or

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e \approx 2.71828\dots$$

### 12.2 Draw the Inverse

Graphically speaking the inverse of a function is a reflection about the line  $y = x$ . The effect is that any point  $(a, b)$  becomes  $(b, a)$  on the inverse graph.

#### Example 3

Graph  $f(x) = e^x$  and its inverse,  $f^{-1}(x)$ .

Press **Y=** enter  $e^x$  into Y1. Press **ZOOM** [4:ZDecimal].



Figure 12. 1



Figure 12. 2

X	Y1
100	2.7048

Figure 12. 3

X	Y1
100	2.7048
10000	2.7181
100000	2.7183
1E6	2.7183
1E7	2.7183
1E8	2.7183
1E9	2.7183

Y1=2.7182818271

Figure 12. 4

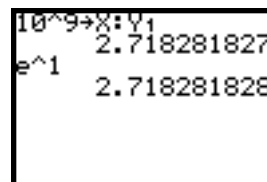


Figure 12. 5

Use the **:** to type two commands.

To draw the inverse:

1. Press **2nd** **DRAW** ; select [8:DrawInv] (see Figs 12.6 and 12.7).
2. Type the function  $e^x$ . Press **ENTER** .  
After pressing ENTER, the calculator automatically goes to the graph screen. (see Fig. 12.8).

**Trouble Shooting:** Since you are in the draw mode you **can not** TRACE on the inverse function. To CLEAR the drawing press **2nd** **DRAW** select [1:ClrDraw].

### 12.1.3 Algebraically Finding the Inverse Function and Graphing $f^{-1}(x)$ .

#### Example 4

Find  $f^{-1}(x)$  for  $y = e^x$  and graph  $f^{-1}(x)$ .

1. Use the definition of logarithm to “undo” the exponential function.  
if  $y = e^x$   
then  $\log_e y = x$
2. Reverse  $x$  and  $y$  to graph the function on the  $x, y$  coordinate plane:  
 $y = \log_e x = \ln x$   
Using function notation:  
if  $f(x) = e^x$   
then  $f^{-1}(x) = \ln x$ .
3. Graph  $y = \ln x$ .

Press **Y=** into Y2; type **LN** **X,T,Θ** (see Fig. 12.9).

Press **GRAPH** (see Fig. 12.10).

**Note:** Now you can TRACE on the inverse graph. The point (1, 0) on graph Y1 is the point (0, 1) on the inverse graph, Y2.

## 12.2 Graphing Common and Natural Logarithms

The graphing calculator has two built-in logarithms:

1. For the common logarithm,  $(\log_{10} x)$ , use the **LOG** key.
2. For the natural logarithm,  $(\log_e x)$ , use the **LN** key.

#### Example 5

Compare the graphs of :

$$f(x) = \log x \quad \text{and} \quad g(x) = \ln x .$$

Enter the functions into Y1 and Y2 then select the window with  $x:[0,4.7]$  and  $y: [-4,4]$  (see Figures 12.11 and 12.12).

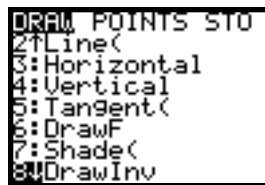


Figure 12. 6

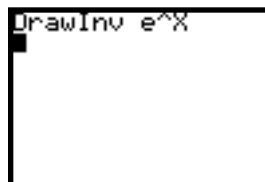


Figure 12. 7



Figure 12. 8



Figure 12. 9

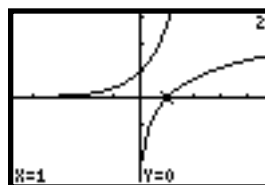


Figure 12. 10



Figure 12. 11



Figure 12. 12

## Interpretation

Since the logarithm is the inverse of the exponential function, the value of the logarithm represents the exponent of an exponential function. Thus, the larger the base of the logarithm the slower the exponent grows. The function  $\ln x$  grows faster than  $\log x$ . The graph of  $\ln x$  is on top of  $\log x$  for  $x > 1$ .

### 12.2.1 Graphs of Logarithms to Bases Other Than 10 and $e$ .

#### Example 6

Find the inverse of  $f(x) = 3^x$  and graph it.

- By definition  $f^{-1}(x) = \log_3 x$ , which is not a built function on the calculator.
- Use Algebra and properties of logarithms to find the inverse function.

$$\begin{aligned}
 y &= 3^x \\
 x &= 3^y && \text{exchange } x \text{ and } y \\
 \log x &= \log 3^y && \text{take the } \log \text{ of both sides} \\
 \log x &= y \log 3 && \text{exponent property} \\
 \log x / \log 3 &= y && \text{divide by } \log 3 \\
 y &= \log x / \log 3
 \end{aligned}$$

**Note:** The change of base formula can be used,  $\log_b N = \log N / \log b = \ln N / \ln b$

- Graph both  $3^x$  and  $\log x / \log 3$ , then compare them.

Press **Y=** enter the functions (see Fig. 12.13)

Press **ZOOM**; select [4:Zdecimal] (see Fig. 12.14). **TRACE** to  $x = 0.7$

- Check numerically:  
 $Y1 = 3^{.7} = 2.15766928$   
 $Y2 = \log 2.15766928 / \log 3 = .7$   
 See Figure 12.15.

Verify the point on the inverse graph.

Press **2nd** **CALC**; select [1:value].

Type in 2.15766928 **ENTER** (see Fig. 12.16).

**V** takes you to the point on the inverse graph (see Figure 12.17).



Figure 12.13

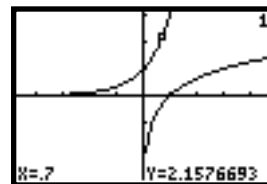


Figure 12.14

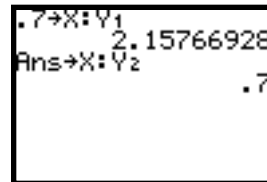


Figure 12.15

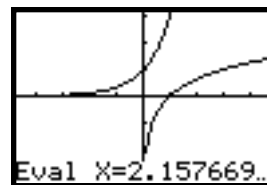


Figure 12.16

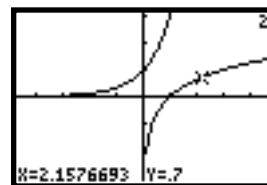


Figure 12.17

### 12.3 Log -log Plots

When data is collected and graphed by hand, log-log paper is sometimes used. We will now explore how power functions behave on log-log plots.

**Note:** Refer back to Chapter 8 for semi-log plots.

#### Example 7

##### 7.1. Graph the power function data and its formula:

L1	1	2	3	4	5	6	7
L2	1	8	27	64	125	216	343

1. Press **STAT**; select [1:Edit]. Enter the data into L1 and L2 (see Fig. 12.18). Notice that  $L2 = L1^3$ . A function that would describe this relation is  $y = x^3$ .
2. Into **Y=** type  $x^3$  (see Fig. 12.19).
3. Set up the plot (see Fig. 12.20). Press **ZOOM**; select [9:ZoomStat].

##### 7.2 Graph the data as a log-log plot.

1. Let  $\log L1 = L3$  and  $\log L2 = L4$  (see Fig. 12.22)
2. **CLEAR** **Y=**
3. Set up the plot with L3 and L4 (see Fig. 12.23)
4. Press **ZOOM**; select [9:ZoomStat] (see Fig. 12.24).

**Note:** Power functions appear to be linear when graphed on a log-log plot.

Verify algebraically:

$$y = x^3$$

$$\log y = \log x^3$$

$$\log y = 3 \log x$$

When you graph this equation,  $\log x$  is the independent variable (input value) and  $\log y$  is the dependent variable (output value). Thus we have a direct proportion with the 3 representing the slope of a line. Thus the slope will always be the power of the function.

**Note:** When data is collected, to test if the data has a power function relationship graph the  $\log$  of list 1 with the  $\log$  of list 2. If the graph appears linear the relationship is a power function. With the slope indicating the power of the function.

Figure 12. 18

Figure 12. 19

Figure 12. 20

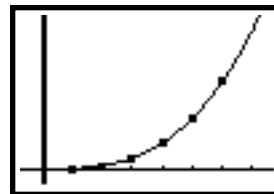


Figure 12. 21

Figure 12. 22

Figure 12. 23

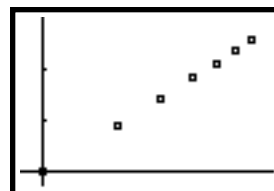


Figure 12. 24