Chapter 11
Quadratic and Polynomial Functions

11.1 Quadratic Functions

Functions of the form $y = ax^2 + bx + c$ are called quadratic or second degree equations.

Note: The highest power of x is two.
All quadratic equations have the shape of a parabola.

Example 1
Graph $y = x^2 - x - 6$ and identify a, b, and c.
For this quadratic equation:

$a = 1$, $b = -1$ and $c = -6$

To graph press [Y=] [CLEAR] . Into Y1 enter $x^2 - x - 6$. Press [ZOOM] [6] , since a=1 the graph opens up (see Fig. 11.1).

Figure 11. 1

11.1.1 Finding y-intercept.

Press [TRACE] to find the y-intercept; if $x = 0$ then $y = -6 = c$ (see Fig. 11.2).
The y-intercept is (0, -6) = (0, c).

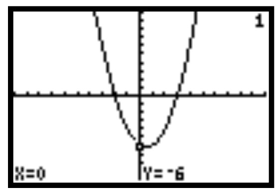


Figure 11. 2

11.1.2 Finding x-intercepts / Finding the Roots/ Finding the zeros.

Use [TRACE] and arrows to estimate where the graph crosses the x-axis. There are two ***x*-intercepts** (where $y = 0$), also known as ***roots*** or ***zeros.*** The left intercept is around $x = -2$ and the right intercept is around $x = 3$. (see Figs. 11.3 and 11.4).

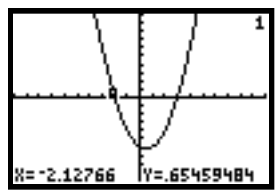


Figure 11. 3

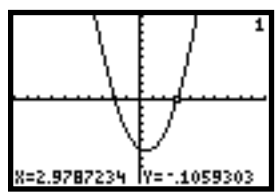


Figure 11. 4

Use the calculator to find the left x-intercept (see Fig. 11.5) by following the steps below (see Figs. 11.6 -11.11 for TI-82, TI-83 differences).

Press [2nd] [CALC] [2] . Follow these steps:

1. Position the cursor **to the left** of the x -intercept when prompted. Press [ENTER]
2. Reposition the cursor **to the right** of the x -intercept when prompted. Press [ENTER] .
3. When prompted GUESS? position the cursor near the x -intercept. Press [ENTER] . The root or x -intercept value is displayed (see Fig. 10.5).

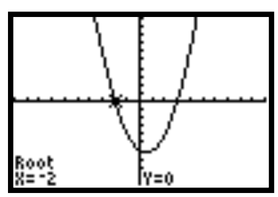


Figure 11. 5

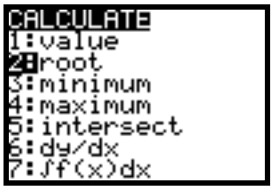


Figure 11. 6 TI-82 CALC Menu

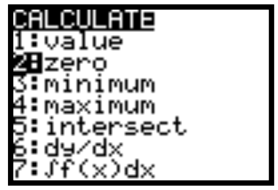


Figure 11. 7 TI-823CALC Menu

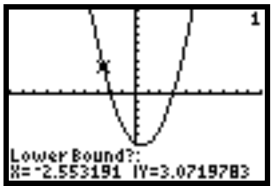


Figure 11. 8 TI-82 "Left" Prompt

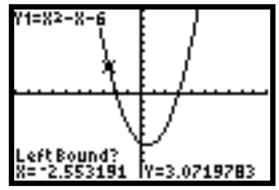


Figure 11. 9 TI-83 "Left" Prompt

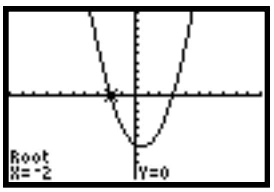


Figure 11. 10 TI-83 "Root"

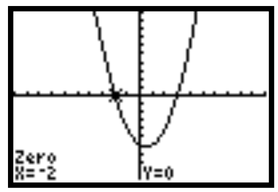


Figure 11. 11 TI-83 "Zero"

Find the right x - intercept:
Repeating the process we find that the other x - intercept is (3, 0)(see Fig. 11.12). There are two roots, at $x = -2$ and $x = 3$.

11.1.3 Check Roots Algebraically:

$y = x^2 - x - 6$

if $x = -2$, $y = (-2)^2 - (-2) - 6 = 0$

if $x = 3$, $y = 3^2 - 3 - 6 = 0$ (see Fig. 11.13).

11.1.4 Finding the Vertex of a Quadratic Function

We see from the graph in Figure 11.12 that the function $y = x^2 - x - 6$ opens upward. This will be true when $a > 0$. As we move left to right the ***minimum*** point on the graph is called the ***vertex.*** It is somewhere between $x = 0$ and $x = 2$.

Example 2

Find the vertex for $y = -2x^2 - 12x - 13$.

Notice that $a < 0$ and the graph opens downward (see Figs. 11.14 and 11.15). Now the vertex is the ***maximum*** point on the graph. The vertex appears to be around the point (-3, 5).
Use the calculator to find the vertex.

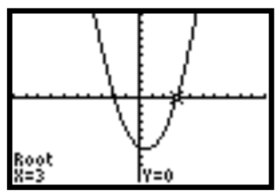


Figure 11. 12

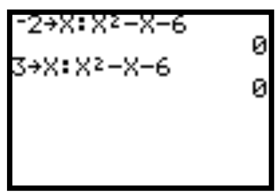


Figure 11. 13

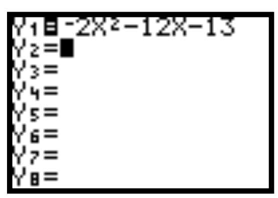


Figure 11. 14

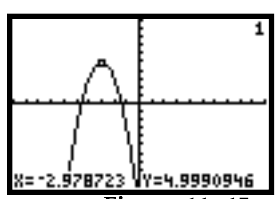


Figure 11. 15

Press [2nd] [CALC] [4] for maximum (see Fig. 11.16). Then follow these steps*:

1. Position the cursor ***to the left*** of the maximum (lower bound) when prompted. Press [ENTER] (see Fig. 11.17).
2. Reposition the cursor ***to the right*** of the maximum (upper bound) when prompted (see Fig. 11.18). Press [ENTER]

When prompted GUESS? position the cursor near the maximum. Press [ENTER] (see Fig. 11.19).

The vertex is at the point (-3, 5) (see Fig. 11.20).

> *<u>**TI-83 Note**</u>**:** As with the x - intercept the TI-83 prompts for left and right bounds. You can also enter a number guess for the maximum. Only the TI-82 screens are shown to the right.

11.1.5 The Vertex Form of a Quadratic Function.

We write the vertex as the point (h, k). Then the quadratic equation $y = ax^2 + x + c$ is transformed to :

$$y = a(x - h)^2 + k$$

So $y = -2x^2 - 12x - 13$ becomes

$$y = -2(x - {}^{-}3)^2 + 5$$
$$= -2(x + 3)^2 + 5$$

Verify algebraically:

$$y = -2(x + 3)^2 + 5$$
$$= -2(x^2 + 6x + 9) + 5$$
$$= -2x^2 - 12x - 18 + 5$$
$$= -2x^2 - 12x - 13$$

Verify Numerically:

Enter both equations into [**Y=**] (see Fig. 11.21). Set up a table of values. Press [2nd] [TblSet] (see Fig. 11.22).

Press [2nd] [TABLE] .

Figure 11.23 shows that for all values of x , Y1 and Y2 are equivalent.

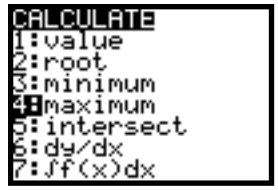


Figure 11. 16

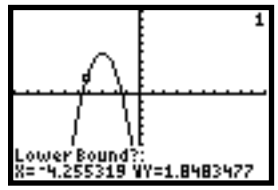


Figure 11. 17

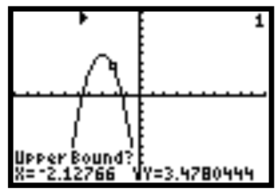


Figure 11. 18

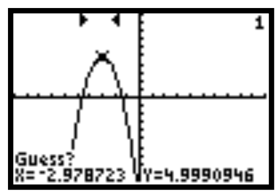


Figure 11. 19

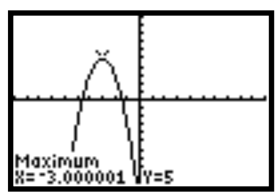


Figure 11. 20

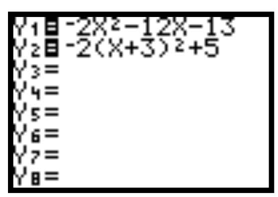


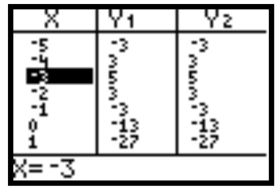
Figure 11. 21

Figure 11. 22

Therefore Y1 = Y2, or

$-2x^2 - 12x - 13 = -2(x + 3)^2 + 5$ and

$ax^2 + bx + c = a(x - h)^2 + k$



X	Y1	Y2
-5	-3	-3
-4	3	3
-3	5	5
-2	3	3
-1	-3	-3
0	-13	-13
1	-27	-27

X= -3

Figure 11. 23

11.1.6 Finding an Appropriate Window.

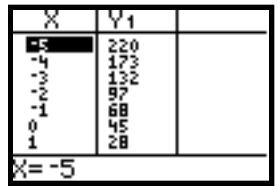
Example 3

Graph $y = 3x^2 - 20x + 45$.

Into Y1 enter: $3x^2 - 20x + 45$.

[CLEAR] all other functions.

Press [ZOOM] [6] . We see nothing!



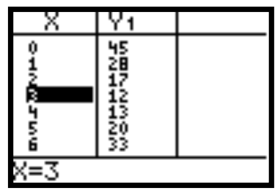
X	Y1
-5	220
-4	173
-3	132
-2	97
-1	68
0	45
1	28

X= -5

Figure 11. 24

Use the table to get a feel for what happens to y as x increases.

Press [2nd] [TABLE] . Figure 11.24 shows that when $x = -5$, $y = 220$ and when $x=0$, $y= 45$. No wonder we couldn't see anything on a [-10,10] by [-10,10] standard window. From algebra we know that $a = 3 > 0$ so the graph opens upward. Use down arrow to find the minimum value of y (the vertex). From the table (see Fig. 11.25) it looks like the minimum occurs around $x = 3$. Adjust the window so that you can see the point (3, 12) as well as the point (-5, 220).



X	Y1
0	45
1	28
2	17
3	12
4	13
5	20
6	33

X=3

Figure 11. 25

Press [WINDOW] [∇] ; let Ymax = 300 (see Fig. 11.26).

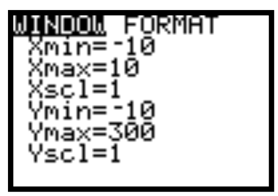


Figure 11. 26

Press [GRAPH] (see Fig. 11.27). It looks like the graph is being cut off on the right side. We need to see more values of x. Press [GRAPH] . Change Xmax to 15.

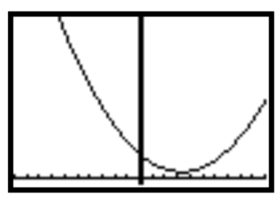


Figure 11. 27

Press [GRAPH] (see Fig. 11.28).

11.1.7 A Complete Graph

Try to select a window that displays a *complete graph*. A complete graph shows the whole shape of the graph, with all its turning points and end behavior. Also shown are the y -intercept and x -intercept(s), if they exist. There are many complete graphs.

Figure 11.28 shows a complete graph of

$y = 3x^2 - 20x + 45$

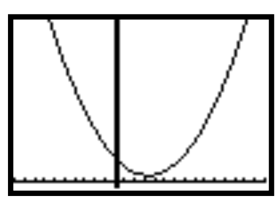


Figure 11. 28

11.2 Polynomial Functions

In Chapter 9 you were introduced to power functions. When positive integer power functions are added together you get a polynomial function:

$y = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x^a + a_0$

where a_n is a constant coefficient ($a_n \neq 0$), and n is a positive integer power.

Example 4

The fall term enrollment of a university's freshman across the years is given below. Find a regression model that would predict the enrollment in 2000 if trends continue

year	freshman
1986	8662
1987	8986
1988	9416
1989	9780
1990	9936
1991	10050
1992	9977
1993	9846
1994	9570
1995	9582
1996	9610
1997	9693

Press STAT [1:Edit]. Enter the years into L1 and the number of freshman in L2 (see Fig. 11.29). Press ZOOM [9:ZoomStat] (see Fig. 11.30). The data appears to increase to 10,050 then decrease to 9,570 then increase again. A polynomial regression equation with at least two turning points is a cubic. Find the cubic regression model.

Press STAT <CALC>; select [:CubicReg] as in Figure 11.31. Type the list names. Press 2nd L1 , 2nd L2 ENTER .

Figure 11.32 shows all the coefficients of the thrid degree polynomial, or cubic function. Store the equation to Y1; press **Y=** VARS select[5:Statistics] <EQ> ; select [:RegEQ] (see Fig. 11.33). Press GRAPH (see Fig. 11.34). To find the predicted number of freshman in 2000, store 2000 to x . The value of Y1 is 10457 freshman in the year 2000 (see Fig. 11.35).

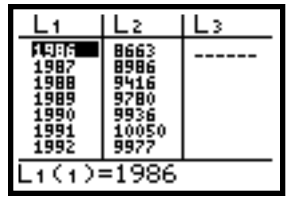


Figure 11. 29

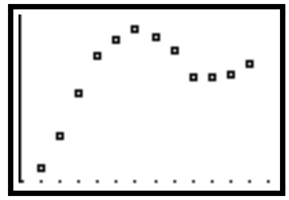


Figure 11. 30

Figure 11. 31

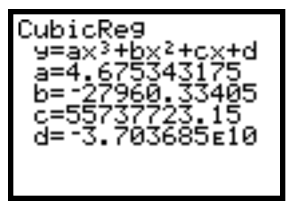


Figure 11. 32

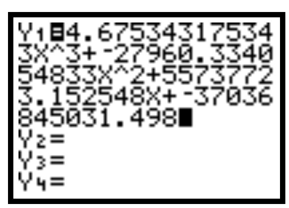


Figure 11. 33

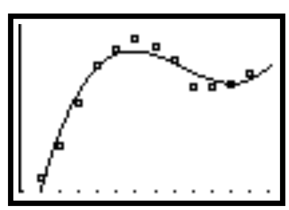


Figure 11. 34

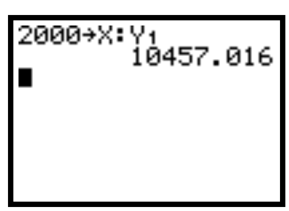


Figure 11. 35