Chapter 9
Power Functions

9.1 Power Functions with Positive Integral Powers

A power Function has the form

$$y = kx^p$$

where k and p are constants. The simplest power function is $y = x$, where $p = 1$. Its graph is linear. However not all other power functions look linear.

Figure 9. 1

9.1.1 Visualizing Odd Power Functions

Example 1

Graph the following and generalize the shape of the graph.

Y1 = x^3
Y2 = x^5
Y3 = x^7

Press **Y=** CLEAR to erase all old functions. Enter the above functions (see Figure 9.1). Set your WINDOW as in Figure 9.2. Press GRAPH (see Fig. 9.3).

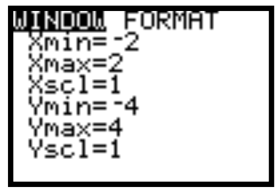


Figure 9. 2

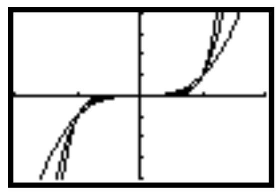


Figure 9. 3

The graphs go through the origin (0, 0), have both positive and negative y values and have a "lazy S " shape. As you move left to right the functions are always increasing.

A look at the table of values, in Figure 9.4, shows that for the odd power functions as x increases to $+\infty$, y increases pretty fast to $+\infty$. As x decreases to $-\infty$ y decreases to $-\infty$.

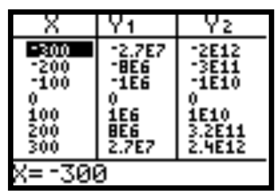


Figure 9. 4

Press ZOOM ; select [3:Zoom Out] (see Fig. 9.5). The calculator gives you a chance to reposition your cursor; then press ENTER (see Fig. 9.6). This shows that all the graphs have a similar shape or "global behavior."

Figure 9. 5

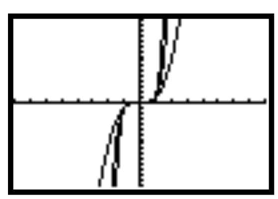


Figure 9. 6

9.1.2 Visualizing Even Power Functions

Example 2:

Graph the following and generalize the shape of the graph.

Y1 = x^2
Y2 = x^4
Y3 = x^6

Enter the functions into [Y=] . Reset the WINDOW as in Figure 9.2 or press [ZOOM] <Memory>; select [1:ZPrevious].

Press [GRAPH] (see Figure 9.7).

The graphs go through the origin (0, 0), have only positive y values and have a "U" shape. As you move from negative values to positive values of x the functions have large positive values then decrease to zero then increase again. A look at the table of values, in Figure 9.8, shows that for the even power functions, as x increases to $+\infty$, y increases pretty fast to $+\infty$. As x decreases to $-\infty$., y increases to $+\infty$.

9.1.3 Polynomial Functions

When positive integer power functions are added together you get a polynomial function:

$y = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x^a + a_0$

where a_n is a constant coefficient ($a_n \neq 0$), and n is a positive integer power.

Example 3

The deer population in a national forest was monitored over a 25 year period. The data collected can be modeled by the fifth-degree polynomial:

$$D(x) = -0.125x^5 + 3.125x^4 + 4000.$$

Graph the function and interpret the graph.

Let Y1= $-.125x^5 + 3.125x^4 + 4000$ (see Fig. 9.9).

> **Note**: Use the table of values to find the appropriate WINDOW.

Press [2nd] [TblSet] . Let TblMin = 0 and Δtbl = 1. Press [2nd] [TABLE] (see Figs. 9.10 and 9.11). It looks like the maximum population occurs around 20 years.

Set the WINDOW as in Figure 9.12.

Press [GRAPH] , then [TRACE] to explore the graph (see Fig. 9.13).

Interpretation:

It took 20 years for the deer population to go from 4000 to a maximum population of about 104,000. Over the next five year period the population decreased back to 4000. The sharp decrease was probably a result of a lack of food supply caused by over population and/or disease.

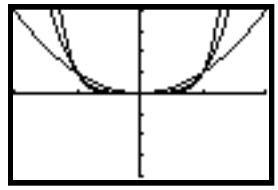
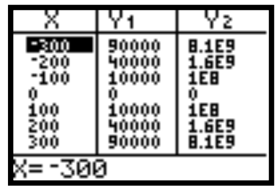


Figure 9. 7

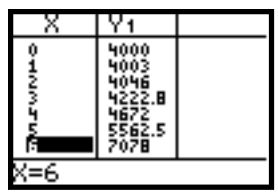


X	Y1	Y2
-300	90000	8.1E9
-200	40000	1.6E9
-100	10000	1E8
0	0	0
100	10000	1E8
200	40000	1.6E9
300	90000	8.1E9

X=-300

Figure 9. 8

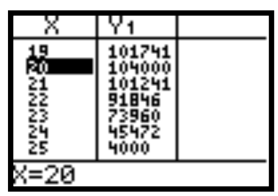
Figure 9. 9



X	Y1	
0	4000	
1	4003	
2	4046	
3	4222.8	
4	4672	
5	5562.5	
6	7078	

X=6

Figure 9. 10



X	Y1	
19	101741	
20	104000	
21	101241	
22	91846	
23	73960	
24	45472	
25	4000	

X=20

Figure 9. 11

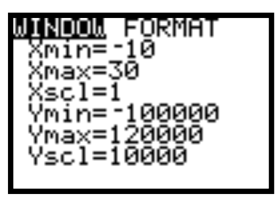


Figure 9. 12

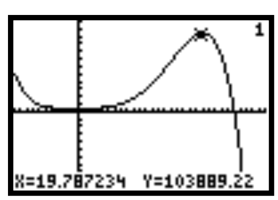


Figure 9. 13

9.1.4 Power Regression Equations.
Example 4
Below are some data reported on AIDS in women. Find a power regression equation that models the data.

Year	AIDS Cases
1	18
2	30
3	36
4	92
5	198
6	360
7	631
8	1016
9	1430

<u>**Note**</u>: Clear functions from Y= and turn OFF all plots.

1. Enter the data in L1 and L2. Press STAT [1:Edit] (see Fig. 9.14). Press 2nd STATPLOT 1 and set up the plot as in Figure 9.15. Press ZOOM ; select [9:ZoomStat] (see Figs. 9.16 and 9.17).
2. Find the regression equation. Press STAT ▷ to <CALC> ; select [B:PwrReg] 2nd L1 , 2nd L2 ENTER (see Figs 9.18 and 9.19).
3. Put the equation into Y1. To paste: *Press Y= VARS ; select[5:Statistics] ▷ ▷ to <EQ> ; select [:RegEQ]. Press GRAPH (see Fig. 9.20).

* <u>**TI-83 NOTE:**</u> Press STAT ▷ to <CALC>. Select [A:PwrReg] , 2nd L1 , 2nd L2 , Y1 to directly store the equation into Y1

The power function fit is pretty good in the beginning, but not so good at the end.

Example 5
Find other polynomial regression equations (quadratic, cubic, or quartic) and an exponential equation .
Repeat steps 1 - 3 above choosing different regression equations. Then use these models to predict future AIDS values. Graphing screens will not be shown.

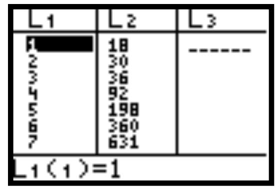


Figure 9. 14

Figure 9. 15

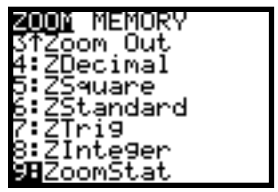


Figure 9. 16

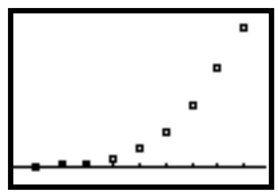


Figure 9. 17

Figure 9. 18

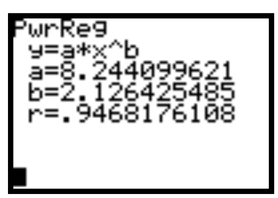


Figure 9. 19

TI-83. Turn DiagnosticsOn under CATALOG to see r.

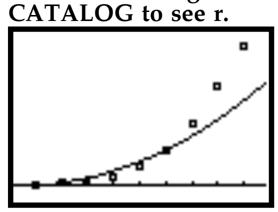


Figure 9. 20

Example 6

Compare the graphs of negative integer power exponents and generalize about odd and even negative integer powers.

First Graph:

Y1 = x^{-1}

See Figure 9.21.

Press [ZOOM] select [4:Decimal] (see Fig. 9.22). This graph is in two pieces, since $x^{-1} = 1/x$, so $x \neq 0$, and it is symmetric to the origin.

Graph other negative odd integer powers:

Y2 = x^{-3}

Y3 = x^{-5}

Enter the functions into Y1 and Y2. See Figure 9.23

Press [GRAPH] (see Fig. 9.24)

To the right of x = 0, the function is at a large negative value and decreases rapidly to zero. As $x \to +\infty$, $y \to 0$.

To the left of x = 0, the function is near zero and decrease rapidly to large negative values. As $x \to -\infty$, $y \to 0$.

Graph even negative interger powers:

Y1 = x^{-2}

See Figure 9.25. Press [GRAPH] (see Fig. 9.26). This is a reasonable graph because $x^{-2} = 1/x^2$, so $x \neq 0$ and all y values are positive. This graph is symmetric to the ***y* -axis.**

Graph other even negative integer powers:

Y2 = x^{-4}

Y3 = x^{-6}

Press [**Y=**] , enter the functions into Y2 and Y3. Press [GRAPH] (see Fig. 9.27).

Once again the functions have the same general shape as x^{-2} with all positive values. On either side of $x = 0$, the function goes to a positive infinity. As one moves left to right the curve begins near zero and increases to +∞. At $x = 0$,, the function is undefined. To the right of zero the function is at +∞ and decreases rapidly to zero.

Troubleshooting: For x^n where n is a decimal value (non-integer) the domain of the function is limited to $x>0$. You will only see one-half of the power function.

Figure 9. 21

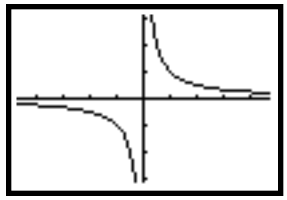


Figure 9. 22

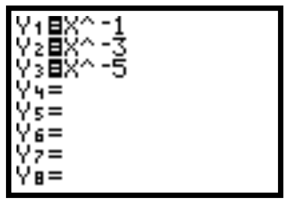


Figure 9. 23

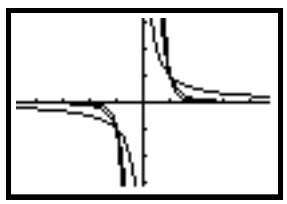


Figure 9. 24

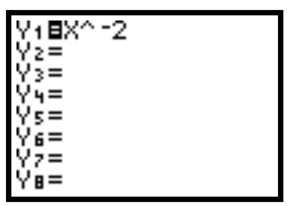


Figure 9. 25

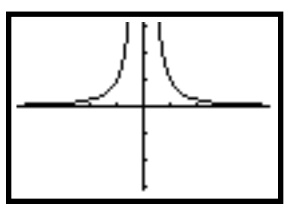


Figure 9. 26

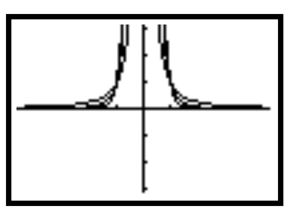


Figure 9. 27