

Chapter 8

Exponential Functions

8.1 Exponential Functions

Exponential functions are in the form $y = b^x$ where $b > 0$, $b \neq 1$ and the power x can be any real number.

Example 1

Graph the following in Y1, Y2, Y3, and Y4:

1. $f(x) = 2^x$
2. $g(x) = 5^x$
3. $h(x) = 7^x$
4. $k(x) = 10^x$

Type the functions into **Y=** (see Fig. 8.1).

Set your **WINDOW** as in Figure 8.2. Press

GRAPH (see Fig. 8.3).

8.1.1 Exponential Growth

All of these graphs cross the y -axis at $(0, 1)$, because $b^0 = 1$. These exponential graphs have the same general shape. As x increases y increases slowly at first and then y increases very rapidly. The value of y is increasing at an increasing rate. This type of change is commonly called exponential growth.

Set up a table of values (see Figs. 8.4 and 8.5). The table gives an idea of the growth rate of $y_1 = 2^x$. As x changes from 0 to 10, Y_1 changes from 1 to about 1000. But as x changes from 10 to 20, Y_1 does not increase to 2000, instead Y_1 now changes from about 1000 to about 1,000,000! This is a magnitude of 3 (3 powers of 10) or 10^3 times as large.

8.1.2 Exponential Decay

Examples 2

Graph the following Y1, Y2, Y3 and Y4:

1. $f(x) = 2^{-x}$
2. $g(x) = 5^{-x}$
3. $h(x) = 7^{-x}$
4. $k(x) = 10^{-x}$

Enter the functions as in Figure 8.6. These exponential functions differ from those in Example 1. A negative now appears in front of the exponent x . Recall $b^{-x} = 1/b^x$. The graphs are shown in Figure 8.7. The graphs from Figure 8.3 are reversed. Now as x increases y decreases very rapidly. This is commonly called exponential decay. Notice that as x gets larger y gets smaller and gets very close to zero. All of these exponential decay graphs have $(0,1)$ as the y -intercept.



Figure 8. 1

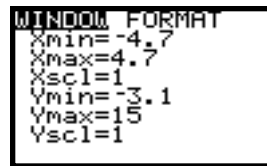


Figure 8. 2

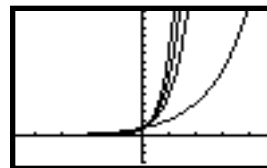


Figure 8. 3



Figure 8. 4

X	Y1	Y2
0	1	1
5	32	3125
10	1024	9.77E6
15	32768	3.1E10
20	1.05E6	9.5E13
25	3.36E7	3E17
30	1.07E9	9.3E20

Figure 8. 5



Figure 8. 6

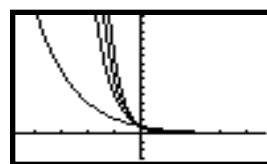


Figure 8. 7

Example 3

The population of a town with a current population of 1100 people is expected to grow at a rate of 5% per year. In how many years will the population double?

When the growth rate is a percent increase, this represents exponential growth. The function is : $P(x) = P_0 (1 + r)^x$. In this case the initial population is $P_0 = 1100$ and the rate of growth is $r = .05$.

Type the function $P(x) = 1100(1 + 0.05)^x$ in

Y= (see Fig. 8.8).

Create a table of values. Press **2nd** **TblSet**, begin with year 0 and set the increment ($\Delta\text{tbl}=\text{=}$) to 1 year (see Figure 8.9). Press **2nd**

TABLE (see Fig. 8.10).

Approximate the doubling time by using the table. Press **▽** until y is approximately 2200, since $2P_0 = 2*1100 = 2200$. When $14 < x < 15$, y will be 2200 (see Fig. 8.11). This means that the population will double in a little more than 14 years.

Example 4

Solve the above problem graphically.

Let $P(x) = 2200$.

Find the solution to:

$$2200 = 1100(1 + 0.05)^x$$

Press **Y=**. Let $Y2=2200$ (see Fig. 8.12).

Adjust the Window based upon the table of values. $x: [-10,20]$ and $y: [-1000,2500]$.

Press **GRAPH**. Find the point of

intersection using the **2nd** **CALC**

[5:intersect] menu (see Fig. 8.13).

Look at the graph of $Y1$ in Figure 8.13. The beginning population of 1100 is the y -intercept of the graph. The point of intersection represents the number of years it will take for the population to double, or y , to become 2200. This will occur when x is about 14.2 years.

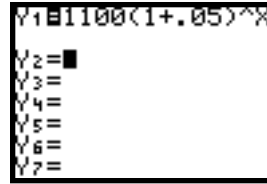


Figure 8. 8



Figure 8. 9

X	Y1
0	1100
1	1155
2	1212.8
3	1273.4
4	1337.1
5	1403.9
6	1474.1

Figure 8. 10

X	Y1
11	1881.4
12	1975.4
13	2074.2
14	2177.9
15	2286.8
16	2401.2
17	2521.2

Figure 8. 11

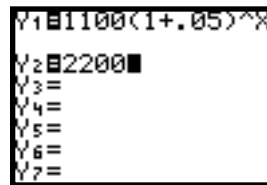


Figure 8. 12

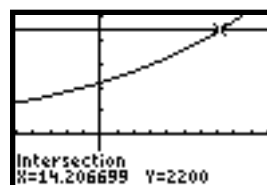


Figure 8. 13

Example 5

Graph the number of E. coli bacteria using a semi-log scale, if the bacteria grow according to the table below, where t is measured in 20 minute time periods.

time periods	number of E. coli
0	200
1	400
2	800
3	1,600
4	3,200
5	6,400
6	12,800
7	25,600
8	51,200
9	102,400
10	204,800

Table 8. 1

- Use the statistics menu to graph the points from Table 8.1. Press **STAT** ; select [1:Edit]; enter the time periods in L1 and enter the number of bacteria in L2 (see Fig. 8.14).
- Find the log of list 2 and store to list 3. Move **Δ** to L3 label. Type **LOG** **2nd** **L2** **ENTER** (see Figs. 8.15 and 8.16).
- Set up the plot. Press **2nd** **STATPLOT** **ENTER** . Set up the plot as in Figure 8.17.
- CLEAR** **Y=** . Press **ZOOM** select [9:ZoomStat] (see Fig. 8.18).

From the data it appears that our E. coli bacteria are growing exponentially because the number is doubling every 20 minutes. Since the number of bacteria is doubling (adding to itself), the growth rate of 100% represents a constant growth factor of 2. Therefore the plot of x against the $\log y$ forms a straight line. So $P = 200(1 + 1)^x$. (See Chapter 12 for more detail).

Note: Exponential data will appear linear when graphed on a semi-log scale

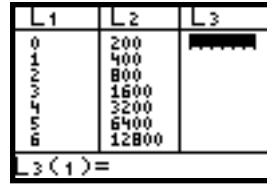


Figure 8. 14

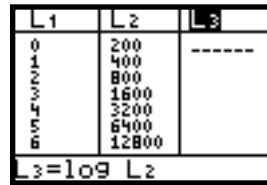


Figure 8. 15

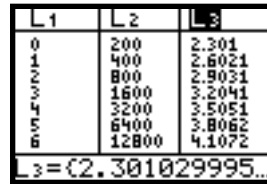


Figure 8. 16



Figure 8. 17

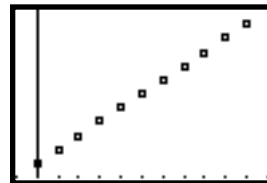


Figure 8. 18

Example 6

Find the Linear regression equation for Example 5 , then give the algebraic explanation.

(You may have to refer to Chapter 12 for the property of logarithms to understand this example.)

Find the linear regression equation using the data in L1 and L3. From the Home Screen press **STAT** <CALC>, select [5:LinReg(ax+b)] (see Fig. 8.19).

Press **2nd** **L1** , **2nd** **L3** **ENTER**

(see Figs. 8.20 and 8.21). The equation is:
 $y = .301x + 2.301$

The algebraic solution will tie the linear regression equation to the exponential function. Use properties of logarithms to simplify:

$$P = 200(2)^x$$

$$\log P = \log (200(2)^x)$$

$$\log P = \log 2^x + \log 200$$

$$\log P = (\log 2)x + \log 200$$

The final equation is in the linear form

$$y = mx + b.$$

$$m = \log 2 = .3010299957...$$

$$b = \log 200 = 2.301299996...$$

These are the same values as a and b in the regression equation (see Fig. 8.22).

Therefore x is the independent value and the $\log P = y$, the dependent value.

We find the equivalent antilog values are:
 $10^{\log 2} = 2$ and $10^{\log 200} = 200$ (see Fig. 8.23).

Therefore the slope of the linear regression equation allows you to find the base of the exponential function and the y-intercept allows you to find the initial value of the exponential function.



Figure 8. 19

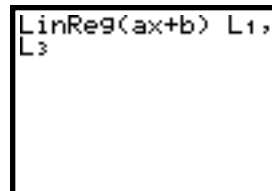


Figure 8. 20

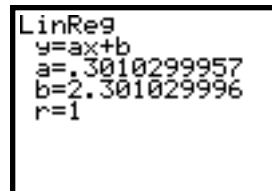


Figure 8. 21

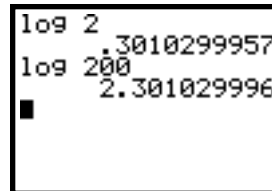


Figure 8. 22

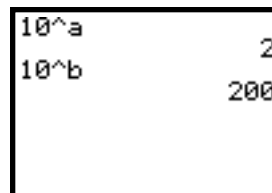


Figure 8. 23

A Special Note to TI-83 Users:

The TI-83 has an addition regression equation that may be helpful in this section. If your data appears to grow exponentially in the beginning, but levels off to its “carrying capacity”, a logistic regression equation might be a good choice. If this kind of data is in L1 and L2, to calculate the logistic regression equation:

Press **STAT** <CALC> [:Logistic] **2nd** **L1** , **2nd** **L2** **ENTER** .