Chapter 7
Scientific Notation and Exponent Properties

7.1 Scientific Notation

Scientific notation expresses large numbers and small numbers using powers of ten : 3250000000 = 3.25×10^9 and 0.00000123586 = 1.23586×10^{-6}. This can be done on the calculator using either [10^x], the power of ten key or [EE], the exponent of ten key.

> **Note**: Any decimal number can be written in scientific notation using the form $K \times 10^x$, where $1 \le K < 10$ and x is an integer. To change back to decimal number form: if $x > 0$ move x decimal places to the right, if $x < 0$ move x decimal places to the left.

Example 1:
Type the following numbers:

1. **$3.25 \text{ X } 10^9$**
2. **0.00000123589**

Press 3.25 [2nd] [10^x] 9 [ENTER] , then press 3.25 [2nd] [EE] 9 [ENTER] . Compare the results (see Fig. 7.1 or 7.2). When .00000123589 is typed, it is changed to scientific notation.

> **Troubleshooting:**
> When $x \ge 10$ or $x \le -4$, in 10^x, the number is written in scientific notation.

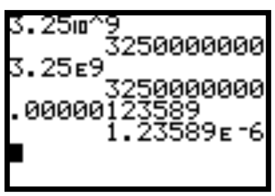

Figure 7.1 TI-82

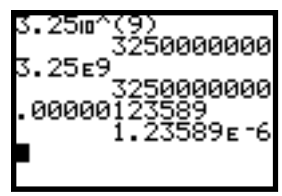

Figure 7. 2
The TI-83 will automatically insert a parenthesis when an exponent is used.

7.2 Verifying Properties of Exponents

Example 2
Verify numerically that the following are true by choosing values for *a*.

1. **$a^0 = 1$**
2. **$a^{-1} = 1/a$**
3. **$a^6 = a \bullet a \bullet a \bullet a \bullet a \bullet a$**

Let ***a*** assume various values. Enter the problems as in Figures 7.3 and 7.4. For the exponent use the [^] key.

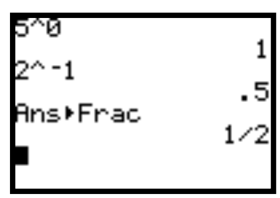

Figure 7. 3

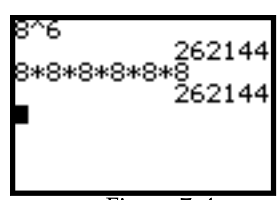

Figure 7. 4

7.2.1 Other Exponent Keys

There are other shortcut keys for exponents. The [x^2] and [x^{-1}] keys are used to paste the exponents without using the [^] key (see Fig. 7.5). The cubic power and radical symbol are found under [MATH] (see Fig. 7.6).

Figure 7. 5

Figure 7. 6

7.2.2 Fractional Exponents

Example 3

Show that the following are equivalent

1. $\mathbf{25^{1/2} = \sqrt{25}}$
2. $\mathbf{32^{1/5} = \sqrt[5]{32}}$

> **<u>Trouble Shooting</u>:** Fractional exponents **must** be enclosed in parentheses.

To access the square root symbol for √25, press [2nd] [√] 25 [ENTER] (see Fig. 7.7).

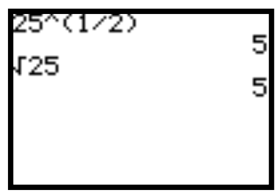

Figure 7. 7

Roots other than square root are found under the MATH menu. To type $\sqrt[5]{32}$, press 5 [MATH] ; select [5: $^x\sqrt{}$] 32 [ENTER] (see Fig. 7.8).

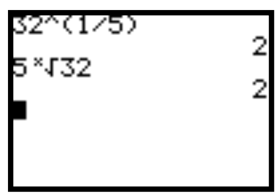

Figure 7. 8

> **<u>Trouble Shooting</u>:**
> The TI-82 has problems raising a negative base to a fractional power other than 1/n. For $a^{m/n}$ where m≠1, the domain is restricted to a≥0. If a<0 you get an **ERR:DOMAIN** message. You have to trick the calculator into performing the operation. The TI-83 does not have this problem. For the procedure to raise -8 to the 2/3 power, see Figures 7.9 and 7.10.

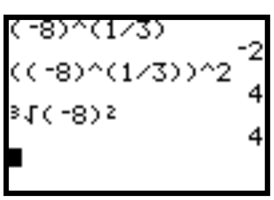

Figure 7. 9 TI-82

The TI-82 does NOT allow (-8)^(2/3), rewrite as a power to a power.

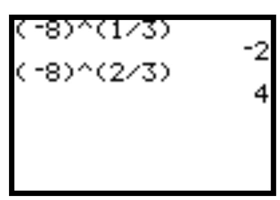

Figure 7. 10 TI-83

The TI-83 accepts (-8)^(2/3).

7.3 Using the Logarithm Key.

To find the exponent or power of ten in an equation, we use logarithms to "undo" the exponent.

If $10^x = N$ then $\log_{10} N = x$

Example 4

Write $10^x = 25$ as a logarithmic equation.

A logarithm is the value of the exponent.
The solution is: $\log_{10} 25 = x$ **.**
You read the above equation as "The logarithm of 25 to base 10 is *x*". To find the value of the exponent press LOG 25 (see Fig. 7.11).

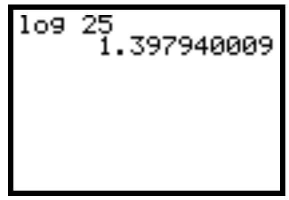

Figure 7. 11

> **Note**: $\log 25 = \log_{10} 25$. This is the common logarithm. The base 10 is understood and conventionally not written.

Check your work:

$10^{1.397940009} = 25$

Press 10 ^ 2nd ANS ENTER

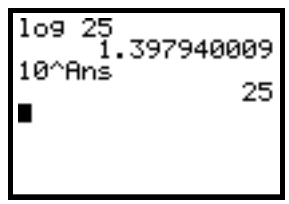

Figure 7. 12

Example 5

Sketch a graph of *y= log x* and use it to determine the domain and range of the function.

Press **Y=** CLEAR LOG X,T,Θ (see Fig. 7.13).

Figure 7. 13

Graph the function. Press ZOOM ; select [4:Zdecimal].

Press TRACE . The graph in Figure 7.14 shows that the ***log x*** is undefined when $x = 0$ (see Fig. 7.14).

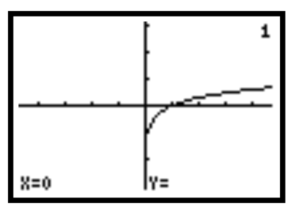

Figure 7. 14

Use ◁ to confirm that ***log x*** is undefined for $x \le 0$ (see Fig. 7.15).

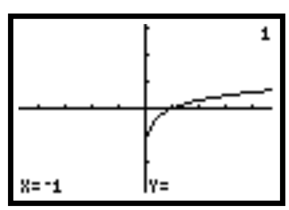

Figure 7. 15

The domain of ***y = log x*** is the set of all ***x*** such that $x > 0$.
The range is evident by looking at the graph also. As ***x*** increases ***y*** increases, but what happens as ***x*** approaches zero? ***y*** seems to be headed in a negative direction.
The range for ***y = log x*** is the set of all ***y*** such that ***y***: $(-\infty, +\infty)$, or ***y*** is any real number.

Verify the range values by using [2nd] [TblSet] (see Fig. 7.16). Use [2nd] [TABLE] to see the values for $y = \log x$ for $0 < x < 1$, y is getting more negative (see Fig. 7.17).

Figure 7. 16

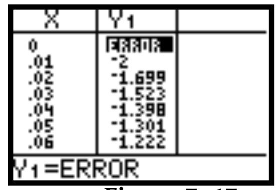

Figure 7. 17

7.4 Solving Equations Graphically

We saw in Chapter 6 that the point of intersection represented the solution to an equation. You can solve an equation graphically by locating the point of intersection.

Example 6

Solve the equation $10^x = 25$ graphically.

Enter the following into **[Y=]** :

$Y1 = 10^x$

$Y2 = 25$

See Figure 7.18.

Figure 7. 18

Set the WINDOW so that both equations can be seen (see Fig. 7.19).

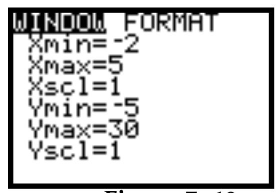

Figure 7. 19

Press [GRAPH] (see Figure 7.20).

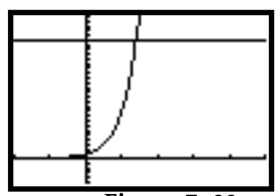
Figure 7. 20

The point of intersection represents the solution to the equation. Press [2nd] [CALC] ; select [5:intersect] (see Fig. 7.21). Follow the prompts by pressing [ENTER] .

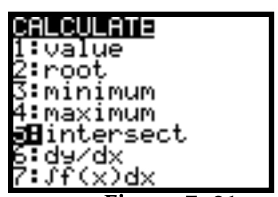

Figure 7. 21

The point of intersection occurs at approximately $x = 1.39794$ (see Fig. 7.22).

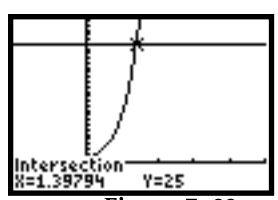

Figure 7. 22

Note: The calculator remembers the intersection value for x. Immediately go to the Home Screen. Press [2nd] [QUIT] ; press [X,T,Θ] .See Figure 7.23 below.

Verify the solution by typing the expression 10^x (see Fig. 7.23 below).

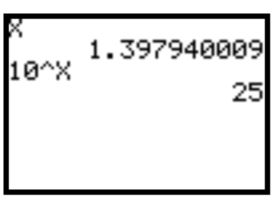

Figure 7. 23