Chapter 6
Intersecting Lines

6.1 Intersecting Lines

Lines that cross each other are said to ***intersect***. All lines eventually intersect unless they are parallel lines or they are the same line.

Example 1

Graph the following two functions and determine the point of intersection, if it exists.

f(*x*) = 3*x* + 4 and g(*x*) = 0.5*x* - 1

Press [Y=] [CLEAR] to clear all the old functions. Enter the above equations into Y1 and Y2 (see Figure 6.1).

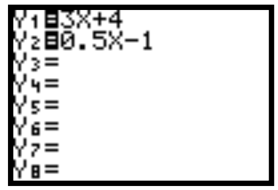

Figure 6. 1

Press [ZOOM] ; select [6: ZStandard] . Press [TRACE] then use [∇] or [Δ] to estimate the point of intersection. It looks like the graphs cross when ***x*** is about -2 and ***y*** is about -2 (see Fig. 6.2).

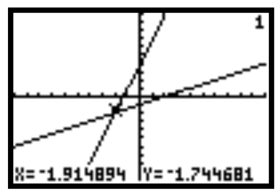

Figure 6. 2

6.1.1 The Calculate Menu

Use the calculator to find the point of intersection.

Press [2nd] [CALC] , select [5:intersect] (see Fig. 6.3).

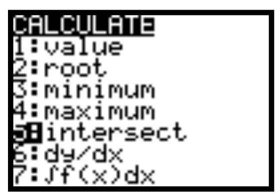

Figure 6. 3

The calculator prompts you:

1. Select the first curve, press [ENTER] .
2. Select the second curve,press [ENTER] .
3. Using the arrows move the cursor near the point of intersection (your guess). Press [ENTER] (see Fig. 6.4 and 6.5).

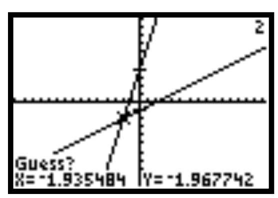

Figure 6. 4

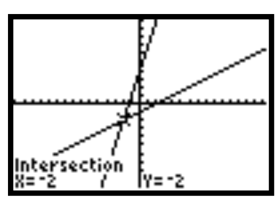

Figure 6. 5

The point of intersection is (-2,-2).

6.1.2 Algebraic Verification

Check algebraically:

If ***x*** = -2 then

Y1= 3***x*** + 4	becomes	Y1 = 3(-2) + 4 = -2
Y2= 0.5***x*** -1	becomes	Y2 = 0.5(-2) - 1 = -2

Both equations have the same ***y*** values so the solution is ***x*** = -2 and ***y***=-2 or the point of intersection is (-2,-2).

6.1.3 Solving Algebraically

Since Y1=Y2 at the point of intersection, you solve algebraically by substitution:

$$\begin{aligned} 3x + 4 &= 0.5\,x - 1 & \\ 2.5\,x + 4 &= -1 & \text{add } -0.5\,x \\ 2.5\,x &= -5 & \text{add } -4 \\ \boldsymbol{x} &= -5/2.5 = -2 & \text{divide by } 2.5 \end{aligned}$$

Substitute $\boldsymbol{x}$ =-2 in Y1 and Y2 as above to find $\boldsymbol{y}$ = -2. The solution is (-2, -2).

6.1.4 Adjusting the Window

Example 2

Find the point of intersection for the system of equations :

h(x) = 0.025x - 25 and

k(x) = -2x + 50

Enter the expressions into Y1 and Y2. Press GRAPH (see Figures 6.6 and 6.7).

The graphs do not appear! Where are they? Press TRACE . This will give you points on the graph (see Fig. 6.8). Using algebra, let $\boldsymbol{x}$ = 0 to find the $\boldsymbol{y}$ - intercepts for the graphs at (0, -25) and (0, 50). To see these points we must choose Ymin and Ymax beyond those points. Set the WINDOW to Figure 6.9. Now we see parts of the graphs. The point of intersection appears to be further to the right (see Fig. 6.10).

Press 2nd TABLE for more information. Y1 is climbing very slowly (m = 0.025) while Y2 is decreasing and somewhere around $\boldsymbol{x}$ = 37 they are about equal (see Fig. 6.11). Experiment with different window settings. Set Xmax so that the point of intersection and beyond can be seen. Press GRAPH and repeat the CALC Menu steps in Section 6.1.1 (see Fig. 6.12).

6.1.5 Solve Algebraically to Confirm

Let Y1=Y2:

$$\begin{aligned} 0.025x - 25 &= -2x + 50 \\ 2.025x - 25 &= 50 \\ 2.025x &= 75 \\ \boldsymbol{x} &= 75/2.025 = 37.0370370... \end{aligned}$$

Substitute the $\boldsymbol{x}$ value into Y1 and Y2. To find the value of $\boldsymbol{y}$.

Y1= 0.025(75/2.025) - 25 =-24.07407407...

Y2= -2(75/2.025) + 50 = -24.07407407...

Although the algebra is pretty fast the arithmetic could still use some help from a calculator!

Figure 6. 6

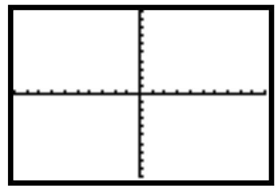

Figure 6. 7

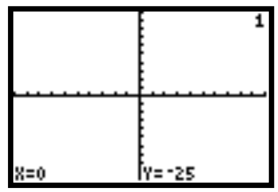

Figure 6. 8

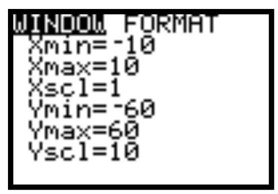

Figure 6. 9

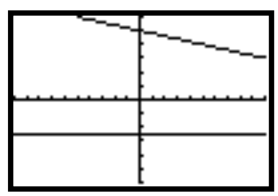

Figure 6. 10

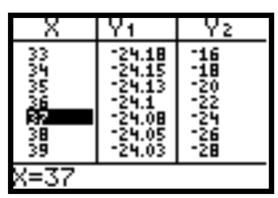

Figure 6. 11

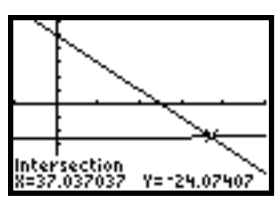

Figure 6. 12

6.2 Piecewise Functions

The graphing calculator can graph piecewise functions very easily. However, it is important to understand how the graphing calculator performs a test.

6.2.1 The TEST Menu

The graphing calculator can tell if a statement is true or false using the TEST menu. Notice that you find the equal and inequality symbols here. Press [2nd] [TEST] (see Fig. 6.13).

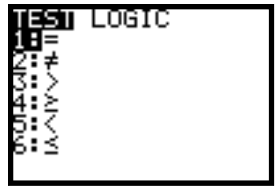

Figure 6. 13

Go to the Home Screen. Press [2nd] [QUIT] .

Example 3
Determine if the following are True or False by typing the following:

- **a.** $5 = 5$
- **b.** $5 \le 7$
- **c.** $5 = 4$
- **d.** $5 \ge 7$

See Figures 6.14 and 6.15.

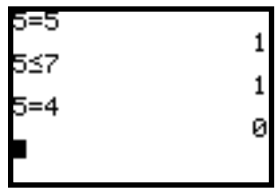

Figure 6. 14

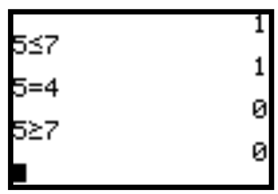

Figure 6. 15

> **Note**: The calculator is performing a test. It tells you **1** for **True** and **0** for **False**.

This method can be used to enter a piecewise function (a function defined in pieces) into the calculator.

Example 4
Graph the piecewise function:

$$f(x) = \quad y = \begin{cases} x + 2 \text{ for } x > 0 \\ -x - 3 \text{ for } x \le 0 \end{cases}$$

y is defined under two conditions:

1. when $x > 0$, use $y = x + 2$
2. when $x \le 0$, use $y = -x - 3$

Press [**Y=**] and enter the equations as in Figure 6.16 .

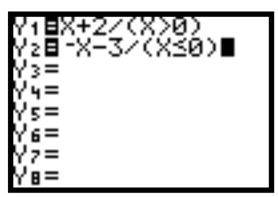

Figure 6. 16

Since this is a piecewise function, if $x > 0$ choose the function $y1 = x + 2$ but if $x \le 0$ choose the function $y2 = -x - 3$ (see Fig. 6.17).

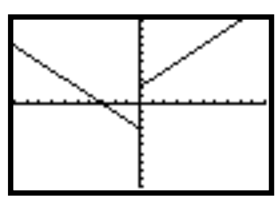

Figure 6. 17

> **Note**: On the calculator we will use the division sign to enter the x value condition. The calculator will perform a test by putting 1 or 0 in the denominator. When the denominator is 0 the function is undefined and no points will be drawn. When the denominator is 1 the function is plotted.

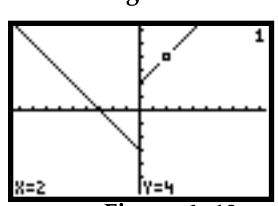

Figure 6. 18

The graph of *f(x)* is in two pieces.
When you are on Y1 you are on the graph of condition one, or ***f(x)* = *x* + 2.**
Press TRACE (see Fig. 6.18).

When you are on Y2 you are on the graph of condition two, or ***f(x)* = -*x* - 3**.
Press ∇ , then TRACE ◁ , to duplicate Figure 6.19. Notice that when the condition no longer applies, no ***y*** value is given. For example if ***x*** = 3 condition two no longer applies (see Fig. 6.20).
To find the value of ***y*** for ***x*** = 3, you must switch to condition one. Δ to the graph of Y1 (see Fig. 6.21).

6.2.2 An Alternate Method for Graphing Piecewise Functions

Many people choose to write a piecewise function all on the same line since ***f(x)*** is defined for all ***x***. This entails making an adjustment to the mode. Press MODE ;
select **Dot** ENTER (see Fig. 6.22).

Example 5
Graph the piecewise function

$$f(x) = \begin{cases} (x+3)^2 - 5 & \text{for } x < -2 \\ 2x + 6 & \text{for } x \geq -2 \end{cases}$$

Press **Y=** CLEAR to clear all expressions.
Type the piecewise function commands as in Figure 6.23. Press GRAPH (see Fig. 6.24).
The advantage of this method is that you can TRACE on the function in the normal manner.

Troubleshooting:

1. If you are in connected MODE the calculator tries to connect the point from the end of one piece of the graph to the end of the other piece, giving the false impression that the function is continuous (see Fig. 6.25).
2. Since you use multiplication for the test, the calculator places a 1 or a 0 inside the test parentheses. This sometimes gives the false value of 0 rather than an undefined value for the function.

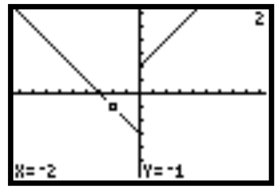

Figure 6. 19

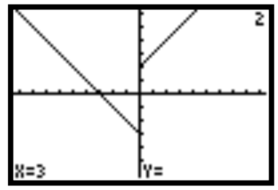

Figure 6. 20

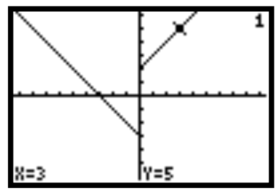

Figure 6. 21

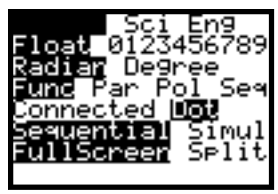

Figure 6. 22

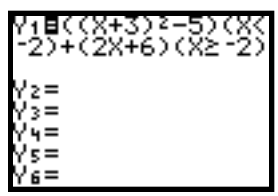

Figure 6. 23 See Troublesooting.

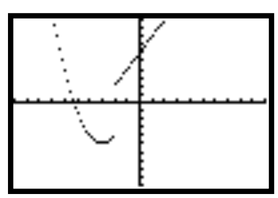
Figure 6. 24 Graph in Dot mode.

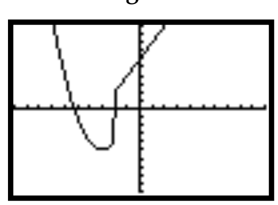
Figure 6. 25
False Graph in Connected mode.

Note: Change back to connected MODE.

Example 6

An 8% flat income tax is represented by *f(x)*. Under the flat tax everyone pays 8% , regardless of how much money is earned per year. A graduated income tax is represented by the piecewise function *g(x)*. Under this plan the first $20,000 is tax free, then between $20,000 and $100,000, 5% tax is paid. If you make beyond $100,000 you pay a 10% tax. Graph these functions.

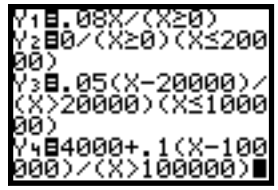

Figure 6. 26

$$f(x) = .08x \quad \text{for } x \geq 0$$

$$g(x) = \begin{cases} 0 & \text{for } 0 \leq x \leq 20000 \\ .05x(x - 20000) & \text{for } 20000 < x \leq 100000 \\ 4000 + .10(x - 100000) & \text{for } x > 100000 \end{cases}$$

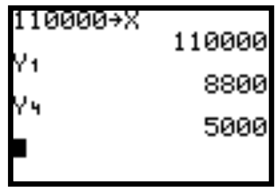

Figure 6. 27

Enter the piecewise functions into the graphing calculator as individual functions with separate conditions.

Enter the function as shown in Figure 6.26.

Y1 = 0.08x /($x \geq 0$)

Y2 = 0/($x \geq 0$)($x \leq 20000$)

Y3 = 0.05(x - 20000) / ($x > 20000$)($x \leq 100000$)

Y4 = 4000 + .1(x - 100000) / ($x > 100000$)

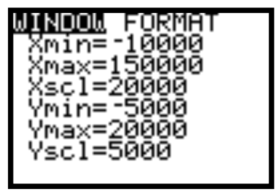

Figure 6. 28

6.2.3 Adjust the Viewing Window

Look at the values for x. They go beyond $100,000. Evaluate Y1 and Y4 for x =110,000 (see Fig. 6.27). With this new information about x and y set the viewing window.

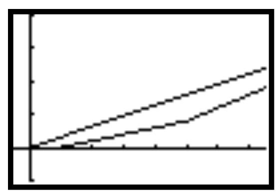

Figure 6. 29

Press [WINDOW] [▽] ; set as in Figure 6.28.

Press [GRAPH] (see Fig. 6.29).

6.2.4 Find the Intersection Point

To see the point of intersection change your WINDOW to Xmax = 350000 and Ymax = 30000. Press [GRAPH] (see Fig. 6.30).

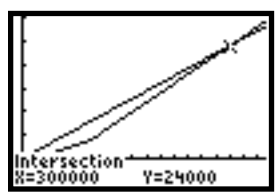

Figure 6. 30

Use [2nd] [CALC] [5:intersect] to find the point of intersection. (Refer back to section 6.1.1 if needed). You will need to select Y1 and Y4 as the pieces of the graphs that intersect (see Fig. 6.30) for the coordinates of the point. This means that under the flat tax system people earning less than $300,000 per year pay more taxes than under the graduated tax plan. The graduated tax is a better system for this group of tax payers.