

Chapter 3

Finding Average Rate of Change

3.1 Finding the Average Rate of Change

Example 1

In 1980 the US Federal debt was 909 billion dollars. In 1990 the Federal debt was 3206 billion dollars. Find the average rate of change.

The average rate of change is:

$$\frac{\text{change in debt}}{\text{change in years}} = \frac{3206 - 909}{1990 - 1980} = 229.7$$

or 229.7 billion dollars per year.

This means on average, the US Federal debt increased by \$229.7 billion/yr. from 1980 to 1990 (see Fig. 3.1).

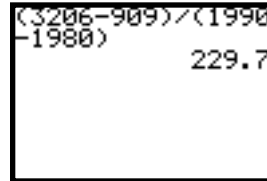


Figure 3. 1

3.1.1 Using Lists to Find the Rate of Change

Enter the data from Example 1 in list 1, L1, and list 2, L2, then calculate the average rate of change.

1. Press **STAT** ; select [4:ClrList];
2. Type: L1, L2, L3 **ENTER** (see Fig. 3.2).
3. Enter the data in L1 and L2. Press **STAT** ; select [1:Edit]. Remember to press **ENTER** or **▽** after every entry (see Fig. 3.3).

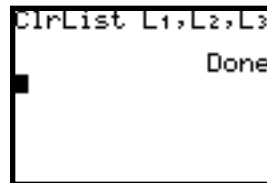


Figure 3. 2

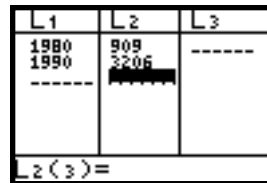


Figure 3. 3

Use the list position to calculate the average rate of change.

Type the following list equation:

$$\frac{(L2(2) - L2(1))}{(L1(2) - L1(1))}$$

Press **2nd** **QUIT** ; then type:

(**2nd** **L2** (2) **-** **2nd** **L2** (1)) **÷**
 (**2nd** **L1** (2) **-** **2nd** **L1** (1))

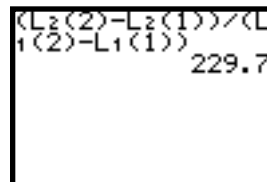


Figure 3. 4
Carefully enter all parentheses.

(see Fig. 3.4). While this may seem tiresome to type, the advantage is that you can change the numbers in the list and then recall the rate of change list equation.

Example 2

In 1985 the federal debt was 1817 billion dollars. Find the average rate of change from 1985 to 1990.

1. Change L1 and L2 to look like Figure 3.5.
2. Recall the rate of change equation on the Home Screen.

Press **2nd** **QUIT** **2nd** **ENTRY** **ENTER**

(see Fig. 3.6).

This means that from 1985 to 1990 the Federal debt increased on average \$277.8 billion/yr.

3.1.2 Working With Longer Lists

Example 3

Below is the Federal debt from 1985 to 1990.

Year	Billions of \$
1985	1817
1986	2120
1987	2346
1988	2601
1989	2868
1990	3206

Note: As previous noted in section 1.1, use the clear shortcut technique to clear lists:

Δ to L1 **CLEAR** **∇** **▶** **Δ** to L2

CLEAR **∇** .

1. Calculate the average rate of change for each year. Enter the above data into L1 and L2. Press **STAT** [1:Edit] (see Fig. 3.7).
2. Create the rate of change equation by using the sequence command and store the values to list 3, L3.

The rate of change equation for any year would be :

$$\frac{(L2(N + 1) - L2(N))}{(L1(N + 1) - L1(N))}$$

Press **2nd** **QUIT** . Type the following:

2nd **LIST**¹, select [5:seq(] (see Fig. 3.8).

Type the commands as in Figure 3.9 and store to list 3, L3.

L1	L2	L3
1985	1817	-----
1990	3206	

L2(3)=		

Figure 3. 5

$(L_2(2) - L_2(1)) / (L_1(2) - L_1(1))$	229.7
$(L_2(2) - L_2(1)) / (L_1(2) - L_1(1))$	277.8

Figure 3. 6

Carefully enter all parentheses.

L1	L2	L3
1985	1817	-----
1986	2120	
1987	2346	
1988	2601	
1989	2868	
1990	3206	

L2(7)=		

Figure 3. 7

OPS MATH
1:SortA(
2:SortD(
3:dim
4:Fill(
5:seq(

Figure 3. 8

seq((L2(N+1)-L2(N))/(L1(N+1)-L1(N)),N,1,5,1)+L3	(303 226 255 26...
---	--------------------

Figure 3. 9

Carefully enter all parentheses

¹ For the TI-83 **2nd** **LIST** <OPS> select [5:seq(] .

Figure 3.10 shows the rate of change values stored in L3. This is a very handy technique for calculating the rate of change for very long lists.

L1	L2	L3
1985	1817	303
1986	2120	226
1987	2346	255
1988	2601	267
1989	2868	338
1990	3206	-----

L1(1)=1985		

Figure 3. 10

Trouble Shooting: The sequence command generates a list of values by evaluating the *expression*, in terms of a *variable*, from a *begin* value to an *end* value by an *increment* value. Thus the command must include the following:

seq(expression, variable, begin, end, increment)

Trouble Shooting: If you want to graph the Rate of Change, graph using L1 and L3. The lists must be the same size. Insert a 0 at the beginning of list 3 in the L3(1) position.

Press **2nd** **INS** 0 (see Fig. 3.11).

L1	L2	L3
1985	1817	0
1986	2120	303
1987	2346	226
1988	2601	255
1989	2868	267
1990	3206	338

L3(1)=0		

Figure 3. 11

Insert zero to make lists the same size.

3.2 Special TI-83 Techniques for Finding the Average Rate of Change of Lists

The TI-83 has a shortcut method for finding the rate of change of a list. Calculate the average rate of change (the difference of elements in L2 divided by the difference of elements in L1), then store it to L3 (see Fig. 3.12).

Press **2nd** **LIST** **▸** to <OPS>, select [7:ΔList(]. Type as in Figure 3.13.

Note: $\Delta\text{List}(\text{list})$ generates a list containing the differences between consecutive elements in a named *list*.

Press **STAT** [1:Edit], to see all three lists (see Fig. 3.14).

See the Trouble Shooting box above Figure 3.11 for graphing the Rate of Change.

NAMES	OPS	MATH
1:SortA(
2:SortD(
3:dim(
4:Fill(
5:seq(
6:cumSum(
7:ΔList(

Figure 3. 12 TI-83 ΔList

ΔList(L2)/ΔList(L1)→L3
{303 226 255 26...

Figure 3. 13 TI-83 Rate of Change

L1	L2	L3
1985	1817	303
1986	2120	226
1987	2346	255
1988	2601	267
1989	2868	338
1990	3206	-----

L3={303, 226, 255...		

Figure 3. 14 TI -83 Lists